A mass addition approach to the bypass turbomachine through flow inverse design problem [†]

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Abstract

In the design of a low bypass ratio fan stage, this paper links the through flow inverse design problems of the fan rotor, the core channel fan stator and the bypass channel fan stator together, as an organic whole, with the fan splitter aerodynamic design problem. It forms thus a unified through flow inverse problem for two-channel turbomachines. While an overall quasi-radial-equilibrium of the streamlines is iteratively achieved among five parts --- the fan rotor, the core channel fan stator, the bypass channel fan stator, the fan splitter and the position of the fan splitter, the blading processes for one fan blade, two fan vanes and one fan splitter ring are performed. This method designs in one pass a fan stage and a splitter with a precise bypass ratio. Its primary and key approach scheme is the introduction of a mass addition flow half way through the through flow streamline development. This scheme can serve as a new feature which can be inserted into those existing through flow programs. When subtracting a mass flow after adding a mass flow, alternatively, this scheme can also be used to treat the inverse design problem of a fan/compressor rotor blade with part-span shrouds.

Keywords: Turbofan; Turbomachine; Streamline curvature; Through flow; Fan splitter; Shrouded rotor blade

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1. Introduction

The streamline curvature through flow inverse design problem on stream surface S2 of a turbomachine is still one of the widely used rational methods for the generation of every kind of turbomachine blade surface, though the state of the art in CFD direct problems is now highly developed. Especially to those researchers and engineers who have less experience, the through flow approach affords a fundamental understanding and a bridge for them to grasp the physical mechanism and the characteristics of the turbomachine flows. As early as Smith (1966) [1] and Novak (1967) [2] formally expressed the full radial-equilibrium equation and the streamline curvature

approach separately, Novak proposed, along with the approach, an important application of this approach --- the through flow design strategy of the system united by a fan component, a splitter and a two-channel flow path in a turbofan engine. The technique he proposed is retold here (Referring to Fig. 1). First, take the through flow design of the bypass channel involving the outer ring inlet, bypass turbomachine and the downstream bypass channel. Second, take the through flow design of the core channel involving the inner ring inlet, lower half blade span and the downstream core channel with or without the core compressors. Finally, perform the iteration between the above two through flow calculations for the convergence about two bounding streamline shapes and two bounding static pressure distributions on the common boundary between the two coaxial flow channels. The necessity to do the iteration is obvious. If the bounding static pressure distribution is not the same, at first
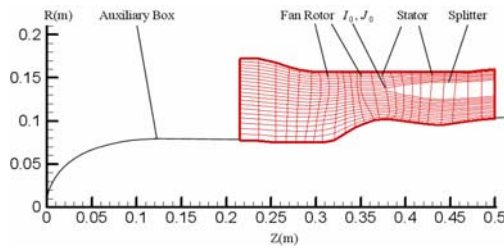
Fig. 1. The bypass turbomachine through flow inverse design problem and its unified through flow design result using the mass addition approach.

the blade surface and the vane surface will have a step type link and second, when the design bypass ratio is realized by outlet pressure control, the flow loss will rise significantly due to streamline distortion.

In the design of a low bypass ratio fan stage, this paper develops a kind of new scheme to directly treat the two-channel through flow inverse problem. As the primary approach, it introduces and controls a mass addition in-between two channels in the through flow model. In this situation a unified quasi-radial-equilibrium is achieved in the inner and outer channel system. It avoids the iteration of two through flow calculations by obtaining the whole blade surface, the two channel shapes and the position of the fan splitter in one pass. Thus a high efficiency and high precision design process is realized. Notice that in all forms of the streamline curvature approach, the momentum equation and continuity equation are separately treated in two sequential steps. So, this mass addition scheme is suitable for different kinds of streamline curvature approach, such as the kind in the concept of circumferential-averaging using radial computing stations (Smith, 1966) [1], the kind in the axisymmetric concept using the radial computing stations (Novak, 1967) [2], the kind in the concept incorporating a mean stream surface $S2_m$ using perfectly the quasi-radial computing stations (Hearsey, 1973) [3] and its further edition with a reformed momentum equation (Hearsey and Wennerstrom, 1975) [4], and so on. The carrier Fortran through flow program here used to check this new scheme is based on the principle of the above kind of 1975 [4].

2. Analysis on the loop model of streamline curvature approach

On the stream surface S2 of an axial flow compressor, the streamline curvature inverse problem utilizes

a 3 layer looping strategy for flow field iteration as mentioned below from inner loop to outer loop. 1. The velocity iteration aimed at the relative velocity (or absolute, for stator) axial components w_z (or w_m). At a streamwise computing station I and on the spanwise streamline J, it integrates, by the implicit method, the quasi-radial-equilibrium equation in the form

$$\frac{\overline{\partial} w_z}{\partial \eta} = A w_z + B/w_z + C \qquad (1)$$

The $\overline{\partial}$ in Eq. (1) represents the derivate of a function on a stream surface. The non-linear functions A, B, C involve w_z itself, the streamline slope σ, the z- and η-wise stream surface derivatives of the prescribed design circulation $c_u r$, the η-wise stream surface derivatives of rothalpy and entropy, the relative velocity circumferential component w_u, the streamline curvature and the stream surface pitch angle and so on in the meridional plane. 2. The mass flow rate iteration for arriving at a prescribed integration value $\dot{m}$. It does mass flow integration at the current computing station I, then changes w_z and integrates again. The continuity equation employed in the streamline curvature approach falls as a mass flow rate equation on each station I

$$\dot{m} = 2\pi K_{\dot{m}} \int_{\eta_{hub}}^{\eta_{tip}} rB\rho \frac{w_z}{\cos\sigma} \cos(\sigma - \theta) d\eta \qquad (2)$$

In it $B(\eta) < 1$ is the blade metal blockage coefficient, and the flow coefficient $K_{\dot{m}}$ is the mass flow ratio associated with the casing boundary-layer blockage, which comes from experience data. 3. The integral flow field iteration to obtain a newly distributed bunch of streamlines. On computing station I, according to the new w_z distribution which affords a new quasi-radial-equilibrium on the older bunch of streamlines and reflects a new mass flow distribution, this iteration interpolates a new spanwise distribution of streamlines which corresponds to the prescribed design mass flow distribution. This interpolation is performed from station $I = 1$ to $I = I_{max}$.

On each computing station the mass flow rate iteration's convergence criteria is the mass flow equals to $\dot{m}$, which is a prescribed turbomachine physical total mass flow. Notice further that ,at first, since the momentum and continuity equations are used in separate

steps, if $\dot{m}$ is a variable along stations, the streamline curvature approach can also converge correctly. Second, in the velocity iteration on one station I, for any given value of $\dot{m}$ which falls in the reasonable scope of the average meridional velocity, the quasi-radial-equilibrium equation will certainly find a corresponding distribution of the axial component w_z (or w_m) on this station. The differences for different $\dot{m}$ are merely different distributions of streamtube thickness and velocity. Based on these criteria, one concludes that it is reasonable to alter the prescribed $\dot{m}$ along a series of computing stations and consequently to alter the prescribed $\Delta \dot{m}$ in different streamtubes. This kind of adjustability permits us the opportunity to control both the streamline shape and the streamtube thickness.

3. The mass addition scheme

Put the outer side tip of the fan splitter mouth at the point (I_0, J_0) of the meridional computing grids (Fig. 1), here the mass flow rate ratio of the channel beyond J_0 to the channel inside J_0 is just the engine's design bypass ratio α. The through flow approach's logic control points which must be reformed are as follows.

(1) When coming into the outer layer loop namely the integral flow field iteration loop, for streamtubes $J = 2$ to $J = J_{max}$, on the base of the accumulating percentage mass flow rate 5%, 10%, ⋯ prescribed by the original through flow approach, re-prescribe a new series of accumulated percentage mass flow rates in those tubes with subscripts $I > I_0$ and $J \geq J_0 - 1$. This series, as an array, has its element values corresponding to the desire of the cross-sectional area contraction of the core and bypass channels. In this paper's example case, the series are defined as an analytical sectional linear function

$$\Delta \dot{m}(I, J) = \frac{J - 1}{J_{max} - 1} + K_{splitter} \times mini\left(1, \frac{I - I_0}{I_{massadd}}\right)$$
$$(I \geq I_0, J \geq J_0) \quad (3)$$

Its first term is just the accumulating percentage mass flow rate of the tube $J - 1$ prescribed by the original through flow approach. $K_{splitter} < 1$ has come to be termed "the splitter casing metal blockage coefficient". Its lowest value boundary can be estimated as the ratio of the splitter's maximum upwind area to the sum of the stator outlet cross-sectional areas of

two channels defined by the turbomachine's overall design. $I_{massadd}$ can be termed as the number of mass addition stations. After $I_{massadd}$ computing stations there will be no further mass addition.

(2) Before the beginning of the middle loop on station I namely the mass flow rate iteration, prescribe the mass addition flow's physical mass flow on station I. For example corresponding to the model Eq. (3) its physical mass flow is

$$\dot{m}_{massadd}(I) = \dot{m}\left[1 + K_{splitter} \times mini\left(1, \frac{I - I_0}{I_{massadd}}\right)\right]$$
$$(I \geq I_0) \quad (4)$$

Eqs. (3) and (4) correspond to the same mass flow. However, Eq. (3) expresses it in percentage form and Eq. (4) expresses $\dot{m}$ in physical or dimensionless physical mass flow form. $\dot{m}_{massadd}(I)$ is used as the convergence criteria of the mass flow rate iteration of station I.

(3) Change the physical mass flow $\dot{m}$ in the proposing formula of the velocity march Δw_z for the I station's next step of mass flow rate iteration as the mass addition flow's physical mass flow, like in the below formula

$$\Delta w_z = \frac{\dot{m}_{massadd}(I) - \dot{m}_{wz}(I)}{\rho(I, J_{middle}) A(I)} \quad (I \geq I_0) \quad (5)$$

Here $\dot{m}_{wz}$ is the integrated physical mass flow rate obtained from the current w_z distribution. Normally ρ value is taken from the central streamline of a channel and A is this channel's cross-sectional area.

(4) Change the physical mass flow $\dot{m}$ in the convergence judging formula of the mass flow rate iteration also as the mass addition flow's physical mass flow $\dot{m}_{massadd}(I)$, $I \geq I_0$.

(5) After the convergence of the I station's mass flow rate iteration, the iteration affords the physical mass flow's new spanwise accumulated distribution $\dot{m}_{wz,J}(I, J)$ for station I. In the integral flow field iteration it serves as the foundation for interpolating the new bunch of streamlines. Now according to the originally prescribed turbomachine physical total mass flow rate $\dot{m}$, count for the accumulating percentage mass flow distribution of this accumulated physical distribution as

$$\Delta \dot{m}_{wz}(I, J) = \dot{m}_{wz,J}(I, J) / \dot{m} \quad (I \geq I_0) \quad (6)$$

Because of the spanwise mass addition, the resulting percentage accumulating distribution $\Delta\dot{m}_{wz}(I,J)$ will exceed 100% from a certain streamtube.

(6) After finishing the integral flow field iteration and before post-processing the data, recover the accumulating percentage mass flow 5%, 10%, ···, prescribed by the original through flow approach for each streamtube, as the first item of Eq. (3).

4. The validation

Fig. 1 shows the through flow inverse problem design result for a test rig with a low bypass ratio, highly loaded, single stage, forward-swept fan component. Its designed mass flow rate is 11.286 kg/s, and stage pressure ratio 2.10, stage adiabatic efficiency 0.85, bypass ratio 0.54, with the rotor diameter 0.314 m and tip velocity 502.6 m/s. Employing an original $S2_m$ streamline curvature approach code and inserting into it the mass addition scheme, this paper performed the fan and splitter's unified through flow design. The flow path of the core channel to be matched by fan stage has been fixed. Thus the flow path of the fan stator for core flow must link the fan rotor path smoothly with the first rotor path of the core compressor. The bypass casing is taken as a cylinder. According to the optimized engine bypass ratio, the outer side tip of the fan splitter mouth is put at the point $I_0=15, J_0=14$ of the meridional grids. Take $I_{massadd}=5$ namely add no more mass flow after station $I=20$. From the estimated initial value $K_{splitter}=0.397$, the design process finally concluded a suitable value $K_{splitter}=0.457$, by which the axial velocity ratios of the stator outlet to stator inlet $c_{z,outlet}/c_{z,inlet}$ for both bypass and core channel stators satisfy the optimizing experience, or to say the fan rotor and stators have reasonable reaction values. This unified through flow design delivers easily a splitter casing with smooth curves and pressure distributions, an accurate bypass ratio and a set of smooth stream surfaces for fan rotor and stators. The real curve of the splitter's inner surface can be extrapolated from streamline $J=13$ according to the distribution rule of the inner channel streamlines. The static pressure distribution about the full domain is shown in Fig. 2 and the through flow design's important numerical results are shown in Table 1. They show that, corresponding to the larger spanwise displacement between two streamlines crossing streamtube 13, there is an evident spanwise static pressure difference.

Table 1. The high loading fan stage's primary variables from Rotor L.E. Station 8 to Stator T.E. Station 20.

| Str	Mach Number Sta. 8	Relative Mach Num 8	Diffusion Factor 13	Mach Number 15	Pressure Ratio 1-20	Static Pressure 20
1	0.3822	0.8114	0.4118	1.0219	2.1995	192364.6
2	0.4302	0.9031	0.5178	0.9770	2.2436	192023.4
3	0.4702	0.9815	0.5697	0.9369	2.2602	191370.3
4	0.5035	1.0504	0.6069	0.8956	2.2564	190391.2
5	0.5319	1.1122	0.6276	0.8572	2.2508	190007.3
6	0.5562	1.1681	0.6317	0.8218	2.2459	189292.5
7	0.5767	1.2192	0.6249	0.7922	2.2496	188546.7
8	0.5944	1.2664	0.6149	0.7656	2.2538	187733.3
9	0.6095	1.3102	0.6049	0.7398	2.2538	86851.0
10	0.6219	1.3506	0.5940	0.7126	2.2476	185878.0
11	0.6322	1.3884	0.5824	0.6856	2.2404	184813.0
12	0.6406	1.4240	0.5699	0.6531	2.2229	183640.3
13	0.6478	1.4578	0.5554	0.6236	2.2143	182341.0
14	0.6546	1.4905	0.5391	0.6079	2.2047	179712.3
15	0.6616	1.5225	0.5271	0.6046	2.2008	180347.9
16	0.6691	1.5541	0.5159	0.6021	2.2008	180833.6
17	0.6769	1.5851	0.5052	0.5985	2.2010	181192.2
18	0.6846	1.6154	0.4966	0.5937	2.2018	181440.6
19	0.6915	1.6448	0.4875	0.5940	2.2129	181597.0
20	0.6967	1.6724	0.4792	0.5955	2.2280	181682.6
21	0.6973	1.6969	0.4661	0.6176	2.2975	181715.7
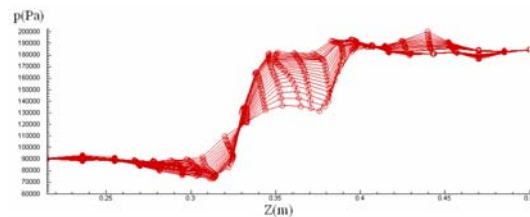
Fig. 2. The static pressure distribution alone the full axial domain and 21 spanwise streamlines of the unified through flow design of a single stage high loading forward-swept fan and its splitter.

5. Concluding remarks

(1) The principle of the through flow velocity iteration tells that the added mass flow has the same aerothermodynamic properties with the original mass flow in the same streamtube. So the mass addition approach does not destroy the solving accuracy of the flow field which still satisfies the radial-equilibrium equation.

(2) By the unified through flow design, one can design and manufacture an one-body fan stator for both the bypass and the core streams for a lower

metal processing cost.

(3) The mass addition approach is valid and very helpful for a low bypass ratio two-channel turbomachine's high efficiency design by avoiding the iteration between two through flow inverse design problems. It is easy to obtain it from reforming any kind of streamline curvature approaches.

Acknowledgment

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